

QCD AND MULTIPARTICLE PRODUCTION – THE STATUS OF THE PERTURBATIVE CASCADE

P. EDÉN

Nordita, Blegdamsvej 17, DK-2100, Copenhagen, Denmark

E-mail: eden@nordita.dk

I discuss recent developments in the QCD cascade formalism. I focus on the importance of and uncertainties in higher order corrections to the Modified Leading Log approximation for final-state radiation. I also talk about the CCFM and LDC evolution equations for initial-state radiation in DIS.

1 Introduction

The QCD cascade is an essential part of our understanding of multiparticle production in the strong interaction. It is the basis of many tests of perturbative QCD. Furthermore, in order to disentangle more and more subtle non-perturbative effects, noting that non-perturbative QCD and the confinement mechanism is one of the main unsolved problems of high energy physics, precise knowledge of the cascade phase is needed.

Thus, there have been activities aiming at improving cascade calculations,^{1,2,3,4,5,6} and I give a brief status report of these in this talk. The content may appear negative and stressing problems, but that is just because I want to focus on unsolved questions that hopefully will be the topics of progress in the near future.

2 e^+e^- annihilation

It is convenient to illustrate the QCD cascade with the logarithmic phase space triangle. With transverse momentum and rapidity expressed in terms of light-cone momenta,

$$k_{\perp}^2 = k_+ k_-, \quad y = \frac{1}{2} \ln\left(\frac{k_+}{k_-}\right), \quad (1)$$

we find that an upper constraint $\sqrt{s}/2$ on the energy $(k_+ + k_-)/2$ implies

$$|y| \leq \ln\left(\frac{\sqrt{s}}{k_{\perp}}\right) + \ln\left(\frac{1 + \sqrt{1 - 4k_{\perp}^2/s}}{2}\right) \approx \frac{1}{2} [\ln(s) - \ln(k_{\perp}^2)]. \quad (2)$$

Thus, the allowed phase space is approximately a triangle in the $(y, \ln(k_{\perp}^2))$ plane, cf. Fig 1. This triangle is suitable for QCD discussions, as the gluon emission density off a $q\bar{q}$ dipole behaves like $dy d(\ln(k_{\perp}^2))$.

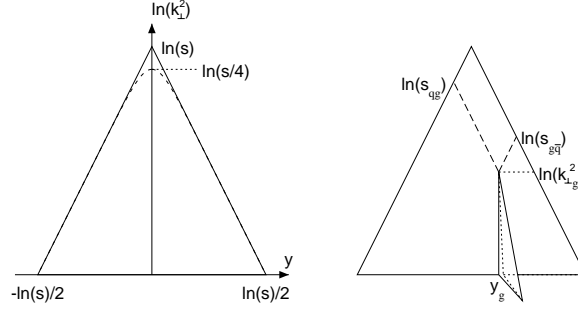


Figure 1. **left:** A triangle in the (rapidity, $\ln(k_\perp^2)$) plane is a good approximation to the phase space given by a maximum energy (dashed hyperbola) and a minimum k_\perp (base line). **right:** After a gluon emission, the total base line of the two new dipole triangles is larger than $\ln(s)$. The additional phase space can be drawn as a double sided fold.

In the dominant case of significantly different k_\perp , the emission density for two gluons off a $q\bar{q}$ pair is well approximated by one factor representing the emission of the first (hardest) gluon g off the $q\bar{q}$ colour dipole, times another factor representing the density for the second (significantly softer) gluon. The latter is given by the summed densities off two independently emitting dipoles, one for the qg dipole, one for the $g\bar{q}$ dipole. (There is also a negative term suppressed by a relative factor $1/N_c^2$ which reduces the colour factor in the quark directions from $N_c/2$ to C_F , but this will be neglected in this simplified discussion.)

The iterative cascade is based on the observation that this factorization generalizes, provided the cascade is strongly ordered in k_\perp . In the parton formulation of QCD cascades,⁷ the emission density off a dipole is split in two terms, representing the emission off each individual parton of the dipole, respectively. Colour coherence then leads to the well-known angular ordering constraint,⁸ which implies that the parton cascade evolves in angle.

After the emission of the first gluon g , the $q\bar{q}$ phase space triangle is not large enough to represent the two new dipoles. In the right half of Fig. 1, the additional phase space is drawn as a double sided fold where the two new triangles meet. The top of this fold will then have coordinates at

$$y_g = \frac{1}{2} \ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right), \quad \ln(k_{\perp g}^2) = \ln\left(\frac{s_{qg}s_{g\bar{q}}}{s}\right). \quad (3)$$

The fold represents the phase space for emissions along the new gluon, with rapidities larger than the latest emission, in other words, the angular ordered cone around the gluon. For each subsequent emission, a new fold is added, and

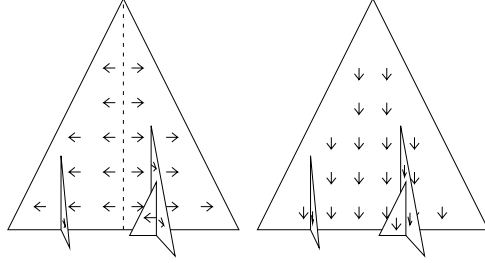


Figure 2. The parton picture of the cascade evolves in angle (left) and the dipole picture evolves in k_{\perp} (right).

thus the triangular picture with folds illustrates the iterative QCD cascade.

Alternative to the parton formulation, the cascade can be interpreted in terms of dipoles, where each gluon emission splits one dipole into two new ones.⁹ The dipole cascade evolves in k_{\perp} , measured in the rest-frame of the emitting dipole, which makes the formalism manifestly Lorenz invariant. As illustrated in Fig 2, the parton and dipole formulations of QCD cascades are similar, though the evolution parameters differ. Within the accuracy of the modified leading log approximation (MLLA),¹⁰ they are identical.¹¹

The MLLA of QCD cascades systematically includes corrections suppressed by a relative factor $\sqrt{\alpha_s}$. It is quite possible to go further and systematically include higher order corrections embedded in the evolution equations, both in the parton^{12,2} and dipole^{3,4} formalisms.

The results from such an exercise must be interpreted with some care. Corrections beyond MLLA accuracy depend on the treatment of a very hard first gluon, and also “moderately ordered emissions”, by which I mean two subsequent gluon emissions with very similar k_{\perp} . For these, the factorization ansatz of the iterative cascade does not hold. Furthermore, one order beyond MLLA accuracy enter energy conservation effects. This implies that each emitted gluon not only adds phase space to the emitting dipole, but also “eats up” phase space in the neighbouring ones. The evolution parameter is then promoted, from a convenient book-keeping tool of the cascade, to a physical assumption about which gluon indeed is being emitted before others. In a sense, we then take the classical cascade formulation of quantum mechanical multiparton production more seriously than we are allowed to.

The study of higher order corrections in cascades is, nevertheless, justified, for a very simple reason. *Corrections one order beyond MLLA are numerically large, for some easily examined observables like inclusive multiplicity distributions.*

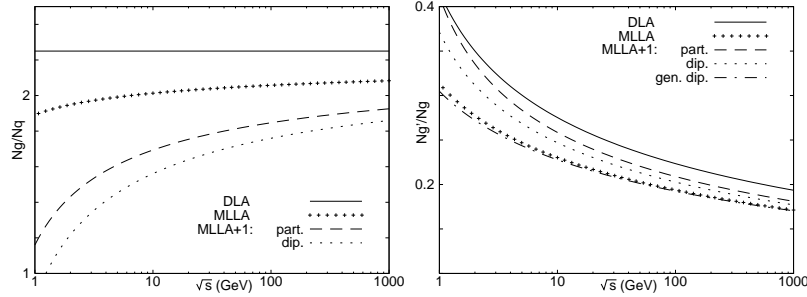


Figure 3. Cascade predictions for the ratio of multiplicities in gluon and quark jets, N_g/N_q (left) and the anomalous dimension N'_g/N_g (right). The leading order result (double log approximation) is shown with solid line, MLLA results with crosses. Corrections one order beyond MLLA are numerically large, and the parton (dashed), dipole (dotted) and generalized dipole (dash-dotted) equations differ noticeably. The two dipole alternatives give essentially identical predictions for N_g/N_q , and only the dipole equation results are shown.

Calculating these corrections, we get an estimate of the important energy conservation effects and, by comparing different cascade pictures, we get a hint to the uncertainties in these estimates.

Fig. 3 shows predictions on the ratio of multiplicities in gluon and quark jets, N_g/N_q , and the anomalous dimension N'_g/N_g . Results are obtained with three alternative cascade evolution equations. One for the parton picture, and two for the dipole picture. The most recent one, called the “generalized dipole evolution equation” is presented in detail elsewhere.⁴ The stronger reduction of N_q/N_g obtained in the dipole equation is favoured by data.^{13,14}

3 Deep Inelastic Scattering

In Deep Inelastic Scattering (DIS), the magnitude of the phase space triangle is determined by the mass W of the hadronic system. The phase space for final state emission off the struck quark is given by the photon virtuality scale Q , and is represented by a lower corner of the total triangle. A virtual propagator, for which $\ln(k_\perp^2) > \ln(k_+ k_-)$, can be represented by a line connecting the k_+ and k_- values at the k_\perp^2 height.

In DIS, high- k_\perp emissions, in particular in the target region, are suppressed. Therefore, the magnitude of the triangle is not as directly linked to the multiplicity as in the corresponding case for a $q\bar{q}$ pair. The picture is nevertheless a very useful illustration of DIS events.

Fig. 5 illustrates how the cascade for initial-state radiation in DIS starts at a cut-off virtuality Q_0 and an energy fraction x_0 (selected from an input

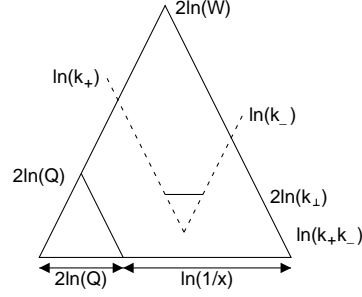


Figure 4. The phase space triangle in DIS. The lower left corner represents the phase space for final-state emissions off the struck quark. The horizontal line represents a virtual propagator with $\ln(k_{\perp}^2) > \ln(k_+k_-)$.

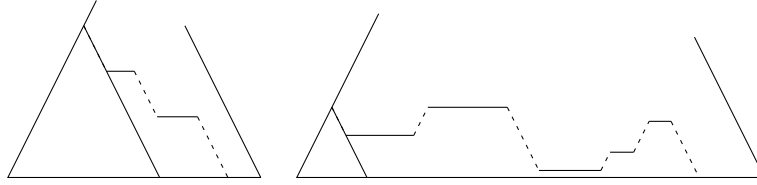


Figure 5. DGLAP (left) evolution dominates for large $\ln(Q^2)$ and moderate $\ln(1/x)$, while non-DGLAP (right) evolution is more important for small $\ln(Q^2)$ and large $\ln(1/x)$. Folds related to emitted partons are not shown.

structure function) and then evolves to the photon interaction point (top of photon triangle). For large Q^2/Q_0^2 and moderate $1/x$, we have DGLAP evolution, where emissions ordered in rapidity are strongly ordered in k_{\perp} .¹⁵ For small Q^2/Q_0^2 and large $1/x$, this need not be the case.

For high enough energies, the BFKL evolution,¹⁶ where emissions are strongly ordered in $\ln(1/x)$ but unordered in k_{\perp} , is expected to dominate. The CCFM⁵ evolution, and the alternative formulation called the “linked dipole chain model” (LDC)⁶, extrapolates smoothly between the DGLAP and BFKL regimes.

Parton cascades in DIS aim at describing both total cross sections and final-state distributions. In the following, I will reduce the final-state distributions to one observable, which is transverse energy in the target direction, called “forward E_{\perp} ”.

Predictions are obtained from evolution equations convoluted with input structure functions at some cut-off scale Q_0 . Fig. 6 shows what happens when Q_0 is raised. The DGLAP evolution predicts no significant forward E_{\perp} , and this simple prediction is of course independent of Q_0 . A change in Q_0 is

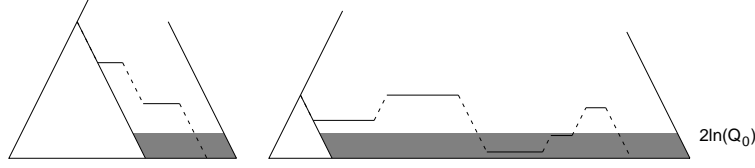


Figure 6. As Fig. 5, but with a higher value of the cut-off scale Q_0 , which implies that perturbative calculations are not trusted in the gray region. In the non-DGLAP case (right), this gives events whose contribution to forward E_\perp is not easily included in the formalism.

compensated by a change in the input parton distributions, leaving the total cross section unchanged.

Non-DGLAP evolution predicts more forward E_\perp , but a quantitative prediction may be difficult to make comfortably insensitive to the cut-off scale Q_0 . With a rise in Q_0 , some previously allowed propagators of the evolution may drop below the cut-off, which in principle implies that we no longer trust their behaviour to be determined by perturbative physics. Thus, it is not clear how to properly treat emission chains like the one above the gray region on the right hand side of Fig. 6, and thus how to properly include their contribution to forward E_\perp .

More investigations are needed to answer whether this contribution is numerically important or not. It may be that the discussion above is a problem in principle only, and not in practice, but it is interesting to note that Monte Carlo investigations of the CCFM and LDC schemes find forward E_\perp to be highly sensitive to subleading corrections in the evolution, and that these corrections move predictions too far below data, while leading order calculations agree rather well.¹⁷

4 Summary

As a natural consequence of the success of the parton cascade formalism, higher order corrections are being examined. These are found to be

- theoretically uncertain,
- numerically important.

This should of course be interpreted like this: QCD in Multiparticle production is an interesting field of physics, suitable for cooperation between experimentalists and theorists!

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